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Drops in the Bucket® Mathematics

RESEARCH FOUNDATION AND INSTRUCTIONAL FRAMEWORK

Supporting Mathematics Achievement Through Retrieval Practice, Spaced Review, Interleaving, and Data-Informed Instruction

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Executive Summary

Drops in the Bucket® Mathematics is a structured mathematics achievement system designed to strengthen mathematical understanding through daily skill review, cumulative practice, spaced repetition, interleaved learning opportunities, and ongoing progress monitoring. The instructional design incorporates multiple principles supported by educational research, including retrieval practice, distributed practice, explicit instruction, formative assessment, and Multi-Tiered Systems of Support (MTSS). Through consistent exposure to previously taught concepts and skills, students develop procedural fluency, conceptual understanding, mathematical reasoning, and problem-solving abilities while reinforcing grade-level standards. The program is designed to complement core mathematics instruction by providing systematic review, targeted intervention support, and opportunities for differentiated learning across diverse student populations. Flexible implementation options support Tier 1 classroom instruction, Tier 2 targeted intervention, and Tier 3 intensive support, making the program adaptable to a variety of instructional settings. By combining research-supported instructional practices with ongoing opportunities for practice and progress monitoring, *Drops in the Bucket*® mathematics is designed to support retention, address learning gaps, and promote long-term mathematics achievement. The program is designed to support standards-based instruction, intervention services, learning recovery efforts, and data-informed decision making while providing educators with practical tools for reinforcing essential mathematical skills and concepts.

Table 1. *Key Research Foundations at a Glance*

Key Research Foundations at a Glance

RESEARCH FOUNDATION	KEY RESEARCHER(S)	MAJOR FINDING	APPLICATION WITHIN DROPS IN THE BUCKET® MATHEMATICS
Retrieval Practice	Roediger & Karpicke (2006); Dunlosky et al. (2013)	Actively retrieving information strengthens retention and improves long-term learning.	Daily review activities require students to recall and apply previously learned mathematical concepts and procedures.
Spaced Practice (Distributed Learning)	Ebbinghaus (1885/1913); Cepeda et al. (2006); Dunlosky et al. (2013)	Learning is retained more effectively when review opportunities are distributed across time.	Mathematical skills are revisited throughout the instructional sequence rather than practiced only during initial instruction.
Interleaving	Rohrer (2012); Dunlosky et al. (2013)	Mixing different problem types improves discrimination, strategy selection, and transfer of learning.	Students encounter multiple mathematical concepts and skill types within a single lesson.
Cumulative Review	Rosenshine (2012); Dunlosky et al. (2013)	Ongoing review of previously learned material promotes retention and reduces forgetting.	Previously taught skills continue to appear throughout the program, reinforcing long-term learning.
Formative Assessment	Black & Wiliam (1998)	Frequent assessment and feedback improve instructional decision making and student outcomes.	Assessment and progress-monitoring resources help educators identify strengths, needs, and growth over time.
Progress Monitoring	Black & Wiliam (1998)	Student performance data can guide interventions and instructional adjustments.	Progress-monitoring tools support data-informed instructional planning and intervention efforts.
Mathematical Reasoning & Problem Solving	National Mathematics Advisory Panel (2008)	Mathematical proficiency includes conceptual understanding, procedural fluency, and problem solving.	Students apply previously learned concepts, analyze problems, and select appropriate solution strategies.
Multi-Tiered Systems of Support (MTSS)	National Mathematics Advisory Panel (2008)	Early identification and targeted intervention improve student outcomes.	Flexible implementation supports Tier 1 reinforcement, Tier 2 intervention, and Tier 3 intensive support.
Evidence-Based Instructional Practices	Every Student Succeeds Act (2015)	Educational programs should be grounded in research-supported instructional practices.	The program incorporates multiple evidence-informed learning principles identified in educational research.

Table 2. *How Research Informs the Instructional Design*

How Research Informs the Instructional Design

Connecting Research Foundations to Program Features

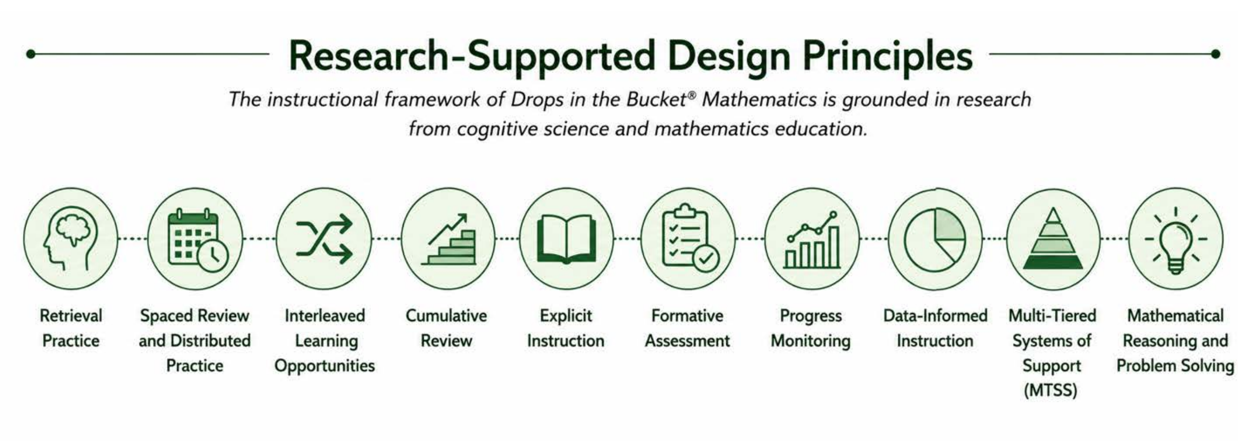
DROPS IN THE BUCKET® MATHEMATICS FEATURE	RESEARCH FOUNDATION	PURPOSE AND INTENDED IMPACT
Daily Review Mathematical skills and concepts are practiced each day.	**Retrieval Practice** (Roediger & Karpicke, 2006; Dunlosky et al., 2013)	Strengthens retention by requiring students to actively recall and apply previously learned mathematical concepts and procedures.
Spiral (Spaced) Review Previously taught skills are revisited throughout the program.	**Spaced Practice / Distributed Learning** (Ebbinghaus, 1885/1913; Cepeda et al., 2006; Dunlosky et al., 2013)	Reduces forgetting and improves long-term retention through review opportunities distributed across time.
Interleaved Practice Different types of problems and skills are mixed within lessons.	**Interleaving** (Rohrer, 2012; Dunlosky et al., 2013)	Improves discrimination, strategy selection, and the ability to transfer learning to new situations.
Cumulative Review Previously learned skills continue to appear throughout the program.	**Cumulative Review** (Rosenshine, 2012; Dunlosky et al., 2013)	Promotes ongoing reinforcement of important skills and reduces the effects of forgetting over time.
Assessments Lesson and unit assessments measure understanding.	**Formative Assessment** (Black & Wiliam, 1998)	Provides frequent feedback to identify learning needs and inform instructional decisions.
Progress Monitoring Ongoing data collection tracks student growth over time.	**Progress Monitoring / Data-Informed Decisions** (Black & Wiliam, 1998)	Supports data-informed instruction and intervention planning to improve student outcomes.

Drops in the Bucket® Mathematics integrates research-supported instructional principles to provide meaningful practice, reinforcement, and feedback—helping students build confidence, strengthen skills, and achieve long-term success.

Research-Supported Design Principles

The instructional framework of *Drops in the Bucket*® mathematics reflects multiple principles identified in cognitive science, mathematics education research, and evidence-based instructional practice. These research-supported principles provide the foundation for the instructional design features discussed throughout this document and have been associated with improved student learning outcomes:

- Retrieval Practice
- Spaced Review and Distributed Practice
- Interleaved Learning Opportunities
- Cumulative Review
- Explicit Instruction
- Formative Assessment
- Progress Monitoring
- Data-Informed Instruction
- Multi-Tiered Systems of Support (MTSS)
- Mathematical Reasoning and Problem Solving



The Need for Daily Mathematics Reinforcement

Mathematics learning is cumulative in nature. New concepts and procedures are built upon previously acquired knowledge, making long-term retention of foundational skills essential for continued academic success. However, educational research has consistently demonstrated that students often struggle to retain newly learned information when opportunities for review and application are limited (Ebbinghaus, 1885/1913). Without systematic reinforcement, students may experience skill decay, gaps in understanding, and difficulty applying previously learned concepts to new mathematical situations.

One of the earliest and most influential findings in the science of learning is the principle that memory weakens over time when information is not revisited. Ebbinghaus (1885/1913) demonstrated that forgetting occurs rapidly following initial learning and that retention improves when information is reviewed and reinforced. Although Ebbinghaus's original work focused on memory rather than classroom instruction, subsequent research has supported the idea that repeated opportunities to retrieve and apply knowledge contribute to stronger long-term retention (Dunlosky, Rawson, Marsh, Nathan, & Willingham, 2013).

These challenges are particularly evident in mathematics, where students are expected to recall and apply previously learned skills while simultaneously acquiring new concepts. A skill that appears mastered immediately after instruction may become less accessible if students are not provided with ongoing opportunities to practice and retrieve that knowledge. As a result, teachers frequently encounter situations in which students require additional review or reteaching of concepts that were previously taught.

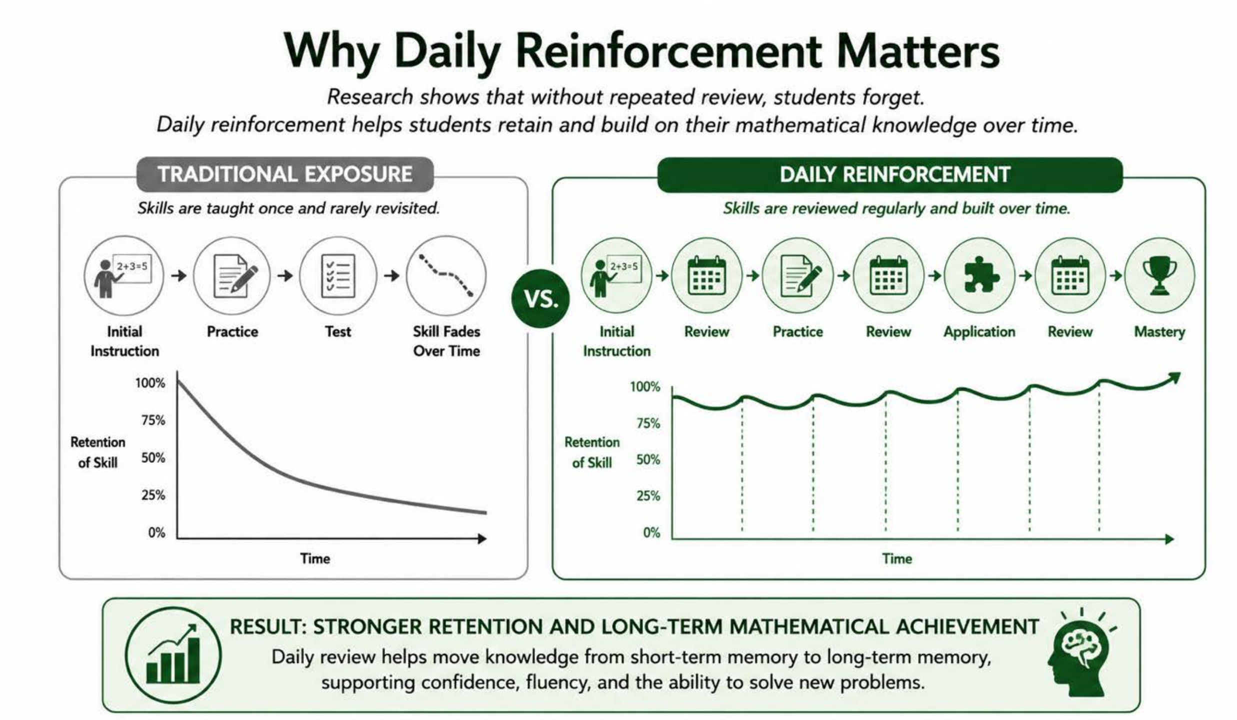
Research examining effective instructional practices has highlighted the importance of sustained engagement with content over time. Hattie (2023) emphasized that student achievement is strengthened when learning experiences include opportunities for practice, feedback, and continued application of knowledge. Rather than viewing learning as a single instructional event, contemporary educational research suggests that mastery develops through repeated interactions with important concepts and skills.

The importance of continued mathematics practice has also been recognized by the National Mathematics Advisory Panel (National Mathematics Advisory Panel, 2008), which identified both conceptual understanding and procedural fluency as essential components of mathematical proficiency. The panel concluded that students benefit from instructional approaches that provide opportunities to develop, strengthen, and maintain mathematical knowledge over time. Ongoing review and reinforcement help ensure that foundational skills remain accessible as students' progress to increasingly complex mathematical content.

Learning gaps often emerge when students do not have sufficient opportunities to revisit previously taught skills. As new concepts are introduced, weaknesses in prerequisite knowledge may become barriers to future learning. Systematic review can help educators identify these gaps and provide additional support before misunderstandings become entrenched. Such approaches are consistent with contemporary frameworks that emphasize early identification of learning needs and targeted intervention (National Mathematics Advisory Panel, 2008).

For these reasons, researchers and educators have increasingly emphasized the value of instructional approaches that incorporate ongoing review and reinforcement. Daily mathematics reinforcement provides students with multiple opportunities to revisit, retrieve, and apply previously learned concepts. These opportunities can strengthen retention, reduce skill loss, support procedural fluency, and contribute to the gradual development of mathematical mastery over time (Dunlosky et al., 2013; Hattie, 2023).

The instructional framework of *Drops in the Bucket*® mathematics reflects these research-supported principles by providing structured opportunities for students to revisit and apply previously taught mathematical concepts on a consistent basis. Through ongoing review and practice, students are encouraged to strengthen retention, build fluency, and maintain access to essential mathematical knowledge as new learning occurs.



Note. This figure illustrates the impact of review frequency on retention of mathematical skills over time. It is informed by research on forgetting and spacing effects (Ebbinghaus, 1885/1913; Cepeda et al., 2006) and the benefits of retrieval practice (Dunlosky et al., 2013).

Instructional Design of *Drops in the Bucket*® Mathematics

Drops in the Bucket® mathematics is designed to provide students with consistent opportunities to revisit, apply, and strengthen previously learned mathematical concepts throughout the school year. The instructional framework is based on the principle that meaningful learning occurs over time and that students benefit from regular engagement with both new and previously taught skills. Rather than concentrating practice within isolated units of instruction, the program incorporates systematic review and reinforcement to support long-term retention and mathematical proficiency.

The program utilizes a structured daily review format in which students complete a brief set of mathematics problems designed to reinforce a variety of skills and concepts. This approach allows students to engage with mathematics on a regular basis while minimizing instructional disruption and preserving time for core classroom instruction. Daily practice opportunities help students maintain access to previously learned knowledge while building connections to newly introduced content.

A distinguishing feature of the program is its spiral review structure. Mathematical concepts are revisited throughout the instructional sequence rather than practiced exclusively during a single instructional unit. Previously taught skills continually appear in subsequent lessons, providing students with ongoing opportunities to retrieve and apply mathematical knowledge. This cumulative approach helps reinforce learning and supports the development of both procedural fluency and conceptual understanding.

The program also incorporates distributed practice by extending opportunities for review across weeks and months of instruction. Skills introduced earlier in the year are systematically revisited, allowing students to strengthen retention and reduce the likelihood of skill loss over time. In addition, lessons include interleaved learning opportunities in which students encounter multiple types of mathematical tasks within a single practice session. This design encourages students to identify appropriate strategies and apply mathematical reasoning across a variety of contexts.

To support data-informed instruction, *Drops in the Bucket*® mathematics includes assessment and progress-monitoring components that help educators identify strengths, monitor growth, and target areas requiring additional support. These resources can be used to inform instructional decisions and guide intervention efforts within a Multi-Tiered System of Supports (MTSS) framework.

The flexible design of the program supports implementation across a variety of instructional settings, including Tier 1 classroom reinforcement, Tier 2 targeted intervention, and Tier 3 intensive support. As a result, educators can adapt the program to meet the needs of diverse learners while maintaining consistent opportunities for mathematical review and practice.

The following sections examine the research-supported instructional principles that inform the design of *Drops in the Bucket*® mathematics, including retrieval practice, spaced review, interleaving, cumulative review, explicit instruction, and formative assessment.

Program Structure and Sample Learning Activities

Drops in the Bucket® mathematics is designed around a consistent daily review format that provides students with repeated opportunities to revisit previously taught concepts while simultaneously engaging with new learning. Each lesson incorporates a variety of skill strands within a single page, allowing students to practice multiple concepts through structured, cumulative review.

The consistent instructional format is intended to reduce procedural demands and increase opportunities for independent practice. Because students encounter familiar lesson structures on a daily basis, instructional time may be focused on mathematical thinking, problem solving, and skill reinforcement rather than repeated explanation of directions and procedures.

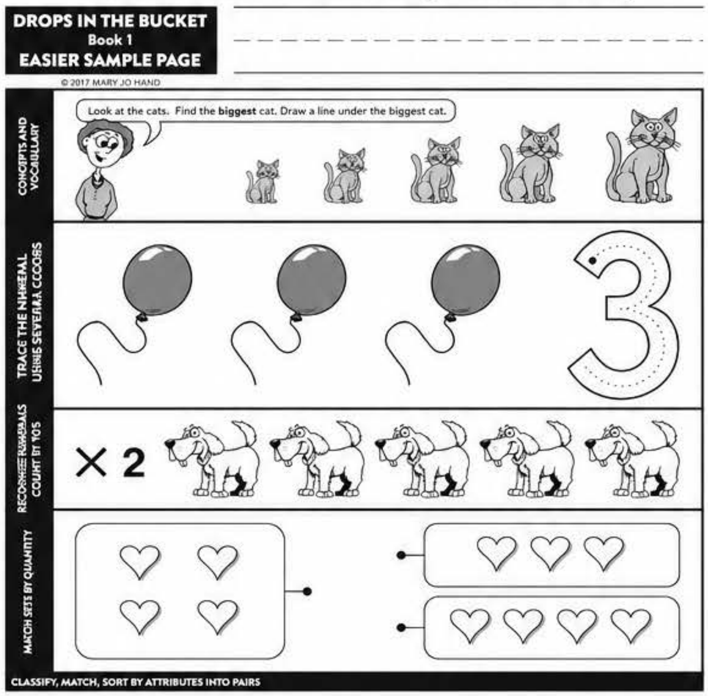
Across grade levels, lessons include opportunities to strengthen number sense, operations, mathematical vocabulary, geometry, measurement, problem solving, reasoning, and other essential mathematical concepts. Skills are revisited systematically over time, providing students with multiple opportunities to retrieve and apply previously learned knowledge.

The sample pages shown below illustrate how the program incorporates cumulative review, distributed practice, retrieval opportunities, and varied mathematical tasks within a single lesson. The examples demonstrate the progression of skills across grade levels while maintaining a consistent instructional framework designed to support long-term retention and mathematical achievement.

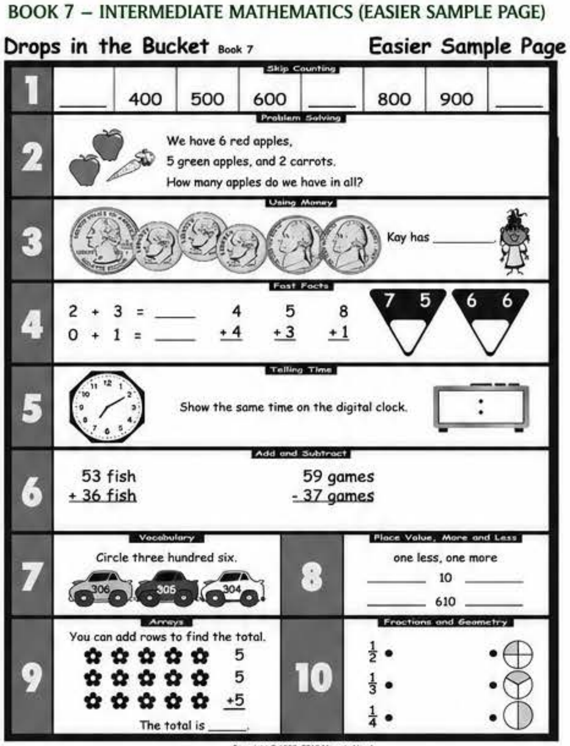
INSTRUCTIONAL DESIGN IN PRACTICE

These sample pages from *Drops in the Bucket®* Mathematics illustrate the consistent daily review format, cumulative practice, and progression of mathematical concepts across grade levels.

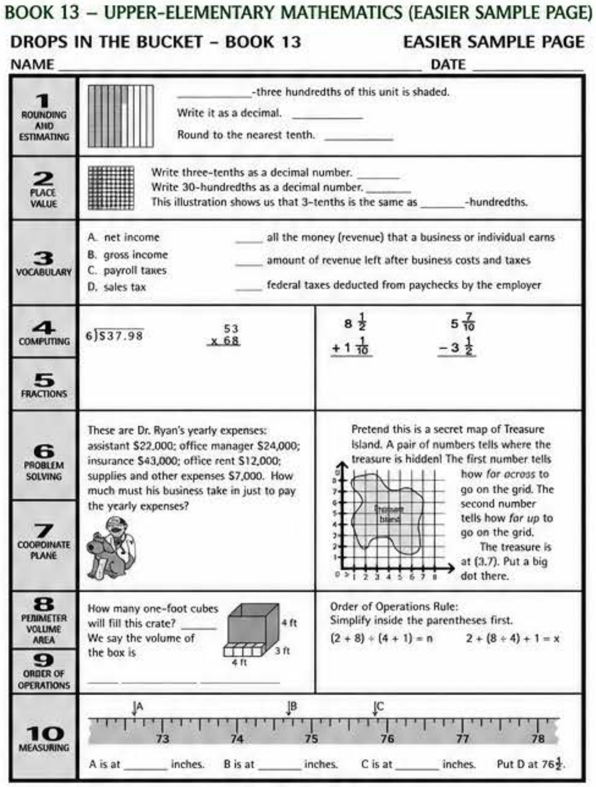
BOOK 1 – PRIMARY MATHEMATICS (EASIER SAMPLE PAGE)



BOOK 7 – INTERMEDIATE MATHEMATICS (EASIER SAMPLE PAGE)



BOOK 13 – UPPER-ELEMENTARY MATHEMATICS (EASIER SAMPLE PAGE)

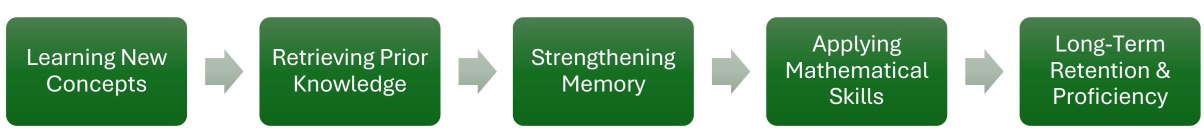


Research Foundation: Retrieval Practice

Retrieval practice is a learning strategy that involves actively recalling previously learned information from memory rather than simply reviewing or rereading content. Research in cognitive psychology has consistently demonstrated that the act of retrieving information strengthens learning, improves retention, and enhances the ability to apply knowledge in future situations (Roediger & Karpicke, 2006). As a result, retrieval practice has become one of the most widely supported evidence-based learning strategies identified in educational research.

Traditional approaches to practice often emphasize repeated exposure to information through review activities. While review remains important, research suggests that learning is strengthened when students are required to actively retrieve information from memory. Roediger and Karpicke (2006) found that students who engaged in retrieval activities demonstrated stronger long-term retention than students who spent an equivalent amount of time studying material without retrieval opportunities. These findings suggest that the process of remembering itself contributes to learning.

Figure 1. *Retrieval Practice and Mathematical Learning*



Note. Figure created by Frog Publications, Inc. based on concepts discussed in retrieval practice research (Dunlosky et al., 2013; Roediger & Karpicke, 2006).

Retrieval practice strengthens learning by requiring students to actively recall and apply previously learned knowledge. Repeated retrieval opportunities contribute to stronger retention, improved transfer of learning, and increased mathematical proficiency.

Dunlosky et al. (2013) identified practice testing and retrieval-based learning as among the most effective instructional techniques available to educators. Their review of educational research concluded that retrieval practice produces robust learning benefits across a variety of content areas, student populations, and educational settings. The researchers noted that retrieval opportunities promote long-term retention and improve students' ability to access previously learned knowledge when needed.

The benefits of retrieval practice are particularly relevant in mathematics instruction. Mathematical proficiency requires students to recall procedures, concepts, vocabulary, number relationships, and problem-solving strategies while simultaneously applying new learning. When previously taught skills are not revisited, students may experience difficulty accessing foundational knowledge needed for more advanced mathematical tasks. Retrieval opportunities help maintain access to this knowledge and support the development of procedural fluency and mathematical reasoning.

The instructional design of *Drops in the Bucket*® mathematics incorporates retrieval practice through its daily review structure. Students are routinely presented with mathematical concepts and skills that have been taught previously and are required to recall and apply their knowledge independently. Rather than focusing exclusively on newly introduced content, students regularly retrieve information from prior learning experiences, strengthening memory pathways and reinforcing mathematical understanding over time.

Because retrieval opportunities occur consistently throughout the instructional sequence, students are encouraged to maintain access to previously learned skills while building connections to new concepts. This ongoing retrieval process supports long-term retention and helps reduce the likelihood that important mathematical knowledge will be forgotten. By embedding retrieval opportunities into daily practice activities, *Drops in the Bucket®* mathematics aligns with research-supported principles associated with durable learning and academic achievement (Dunlosky, et al., 2013; Roediger & Karpicke, 2006).

In mathematics education, learning is most effective when students repeatedly engage with and apply previously acquired knowledge. Retrieval practice provides a mechanism for strengthening memory, reinforcing understanding, and promoting long-term retention. The incorporation of retrieval opportunities within *Drops in the Bucket®* mathematics reflects a research-supported approach to helping students develop and maintain mathematical proficiency over time.

Research Foundation: Spaced Practice and Distributed Learning

Spaced practice, sometimes referred to as distributed practice, is an instructional approach in which learning opportunities are distributed over time rather than concentrated within a single instructional session. Research in cognitive psychology has consistently demonstrated that students retain information more effectively when practice is spaced across multiple learning opportunities rather than massed into a short period of time (Cepeda, Pashler, Vul, Wixted, & Rohrer, 2006). As a result, spaced practice has become one of the most widely supported principles in the science of learning.

The concept of distributed learning can be traced to the work of Ebbinghaus (1885/1913), whose research on memory demonstrated that information is more likely to be retained when learning experiences are repeated over time. Subsequent research has continued to support this principle, suggesting that strategically spaced review opportunities help strengthen memory and reduce forgetting.

In a comprehensive review of the research literature, Cepeda et al. (2006) found that distributed practice consistently produced stronger long-term retention than massed practice. Their findings indicated that spacing learning opportunities across time allows students to retain information more effectively and apply knowledge more successfully in future learning situations. Similarly, Dunlosky et al. (2013) identified distributed practice as one of the most effective learning techniques available to educators, citing substantial evidence supporting its positive impact on long-term achievement.

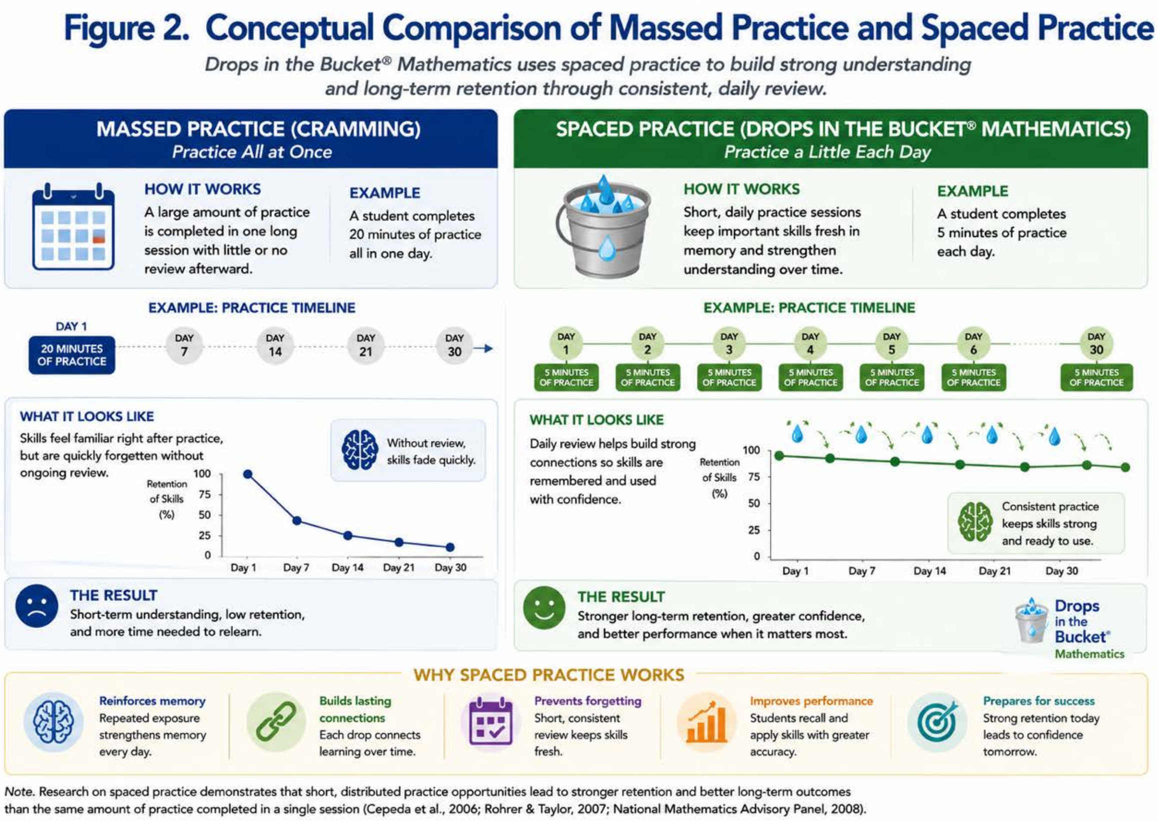
The benefits of spaced practice are particularly relevant in mathematics education. Mathematical learning is cumulative, requiring students to retain foundational knowledge while building increasingly complex skills and concepts. When practice opportunities are limited to a single instructional unit, students may experience difficulty recalling previously learned content when it becomes necessary for future learning. Distributed review helps maintain access to important mathematical knowledge and supports the development of procedural fluency, conceptual understanding, and problem-solving abilities.

The instructional design of *Drops in the Bucket*® mathematics incorporates spaced practice through its systematic review structure. Mathematical concepts are revisited throughout the instructional sequence rather than being practiced only during their initial introduction. Skills taught earlier in the year continue to appear in subsequent lessons, providing students with ongoing opportunities to reinforce learning and strengthen retention. This distributed approach encourages students to revisit previously learned concepts while simultaneously engaging with new mathematical content.

By embedding spaced practice into daily learning opportunities, *Drops in the Bucket*® mathematics aligns with research-supported instructional principles associated with improved retention and long-term achievement. The program's design reflects the understanding that mastery develops through repeated engagement with important concepts over time rather than through isolated instructional experiences.

Research suggests that students benefit when learning opportunities are distributed across weeks and months rather than concentrated into a single period of instruction (Cepeda et al., 2006; Dunlosky et al., 2013). Through ongoing review and reinforcement, spaced practice helps students maintain access to previously learned knowledge and supports the development of durable mathematical understanding.

Figure 2. *Conceptual Comparison of Massed Practice and Spaced Practice*



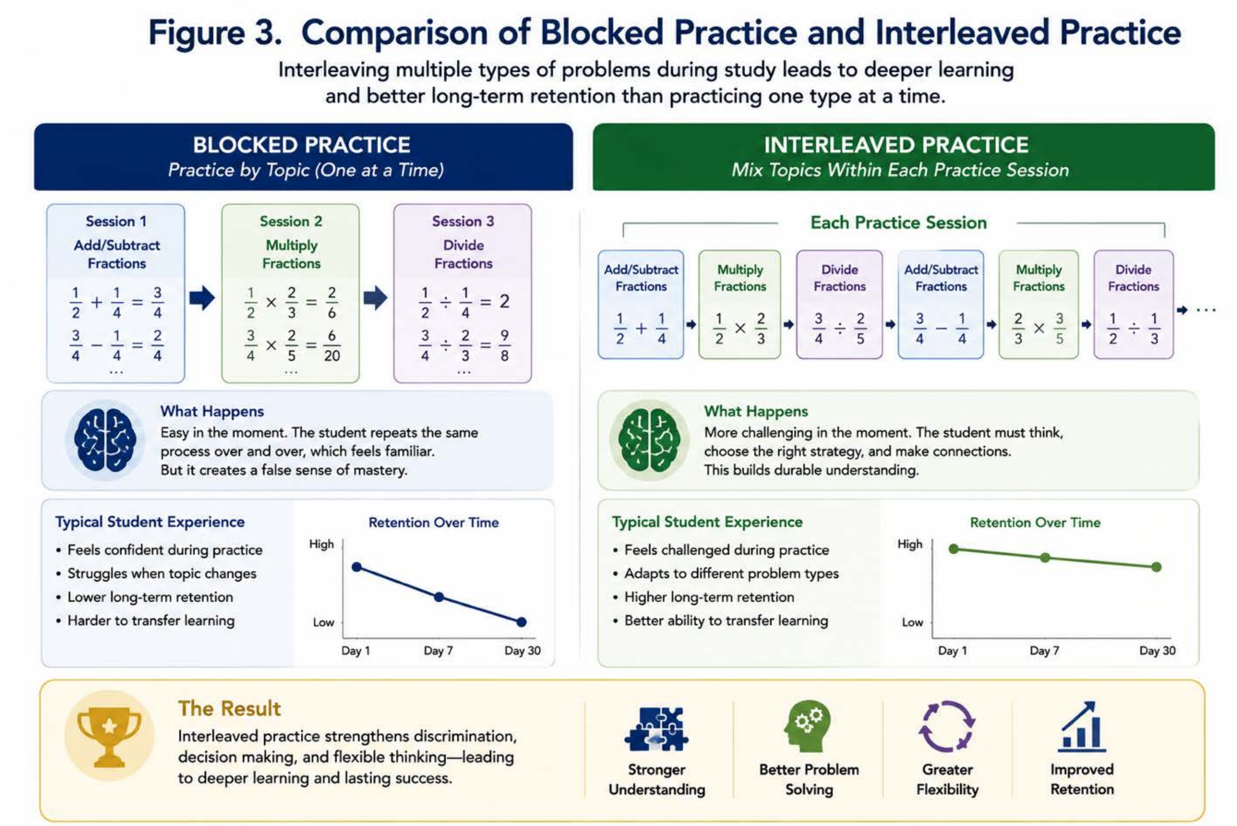
Note. Massed practice concentrates learning opportunities into a single instructional period, whereas spaced practice distributes review and reinforcement across multiple learning opportunities over time. Figure created by Frog Publications, Inc. based on concepts discussed in Cepeda et al. (2006) and Dunlosky et al. (2013).

Research Foundation: Interleaving

Interleaving is an instructional practice in which students encounter multiple types of problems or concepts within a single learning session rather than practicing one skill repeatedly before moving to another. Unlike blocked practice, which groups similar problems together, interleaving requires students to identify the type of problem presented and determine the appropriate strategy needed to solve it. Research has shown that this process can strengthen learning, improve retention, and enhance students' ability to transfer knowledge to new situations (Rohrer, 2012).

Traditional mathematics instruction frequently relies on blocked practice, in which students complete numerous problems requiring the same procedure or strategy. While blocked practice may improve short-term performance during instruction, students often become dependent on contextual cues that signal which solution method should be used. As a result, students may experience difficulty selecting appropriate strategies when confronted with mixed problem types in real-world situations or on assessments (Rohrer, 2012).

Figure 3. *Comparison of Blocked Practice and Interleaved Practice*



Note. Blocked practice groups similar problems together and often encourages repeated use of a single procedure. Interleaved practice presents a variety of problem types, requiring students to identify appropriate solution strategies and apply previously learned knowledge. Figure created by Frog Publications, Inc. based on concepts discussed in Rohrer (2012) and Dunlosky et al. (2013).

Interleaving addresses this challenge by requiring students to discriminate among different mathematical concepts and procedures. Rather than automatically applying the same strategy repeatedly, students must evaluate each problem and determine the most appropriate approach. This process encourages deeper cognitive engagement and supports the development of flexible mathematical thinking. Research suggests that students who engage in interleaved practice often demonstrate stronger long-term retention and improved ability to apply learning across a variety of contexts (Dunlosky et al., 2013; Rohrer, 2012).

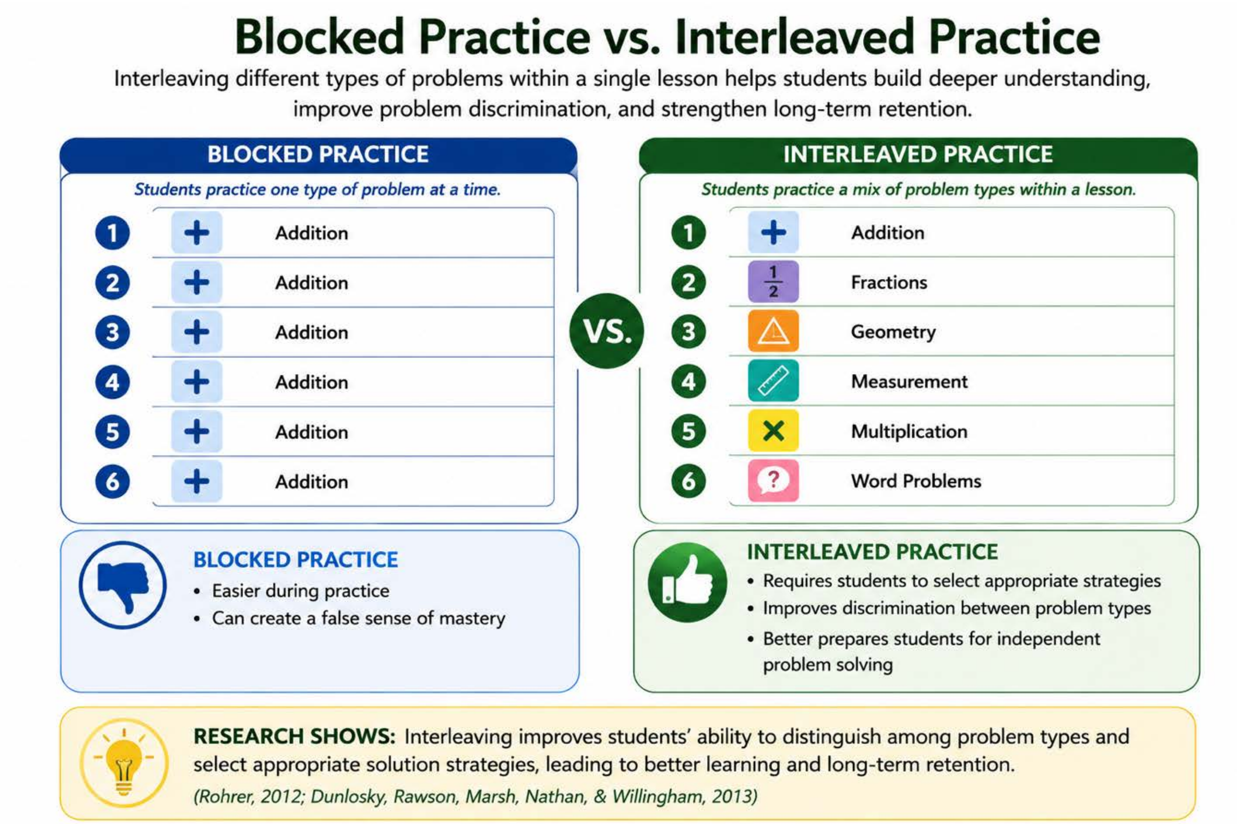
The benefits of interleaving are particularly relevant in mathematics education. Mathematical proficiency requires students to recognize patterns, select appropriate procedures, and apply previously learned concepts to solve unfamiliar problems. Students rarely encounter problems in isolation outside the classroom. Instead, they must draw upon a variety of skills and strategies simultaneously. Interleaved learning opportunities help prepare students for these demands by encouraging strategic thinking and problem-solving across multiple mathematical domains.

The instructional design of *Drops in the Bucket*® mathematics incorporates interleaving through its daily review structure. Students encounter a variety of mathematical concepts within a single lesson, including computation, number sense, geometry, measurement, data analysis, and problem-solving tasks, depending on the grade level and instructional focus. Because multiple concepts are presented together, students must determine which mathematical strategies and procedures are appropriate for each task rather than relying solely on repetition of a single skill.

This approach provides students with regular opportunities to connect previously learned concepts, strengthen mathematical reasoning, and develop greater flexibility in their thinking. By engaging with multiple types of mathematical problems during daily practice, students are encouraged to build a more integrated understanding of mathematics while maintaining access to previously learned knowledge.

Research suggests that interleaving promotes deeper learning by requiring students to actively distinguish among concepts and select appropriate solution strategies (Rohrer, 2012). Through the inclusion of mixed problem types and varied mathematical tasks, *Drops in the*

Bucket® mathematics aligns with research-supported instructional practices associated with improved retention, transfer of learning, and mathematical proficiency (Dunlosky et al., 2013; Rohrer, 2012).

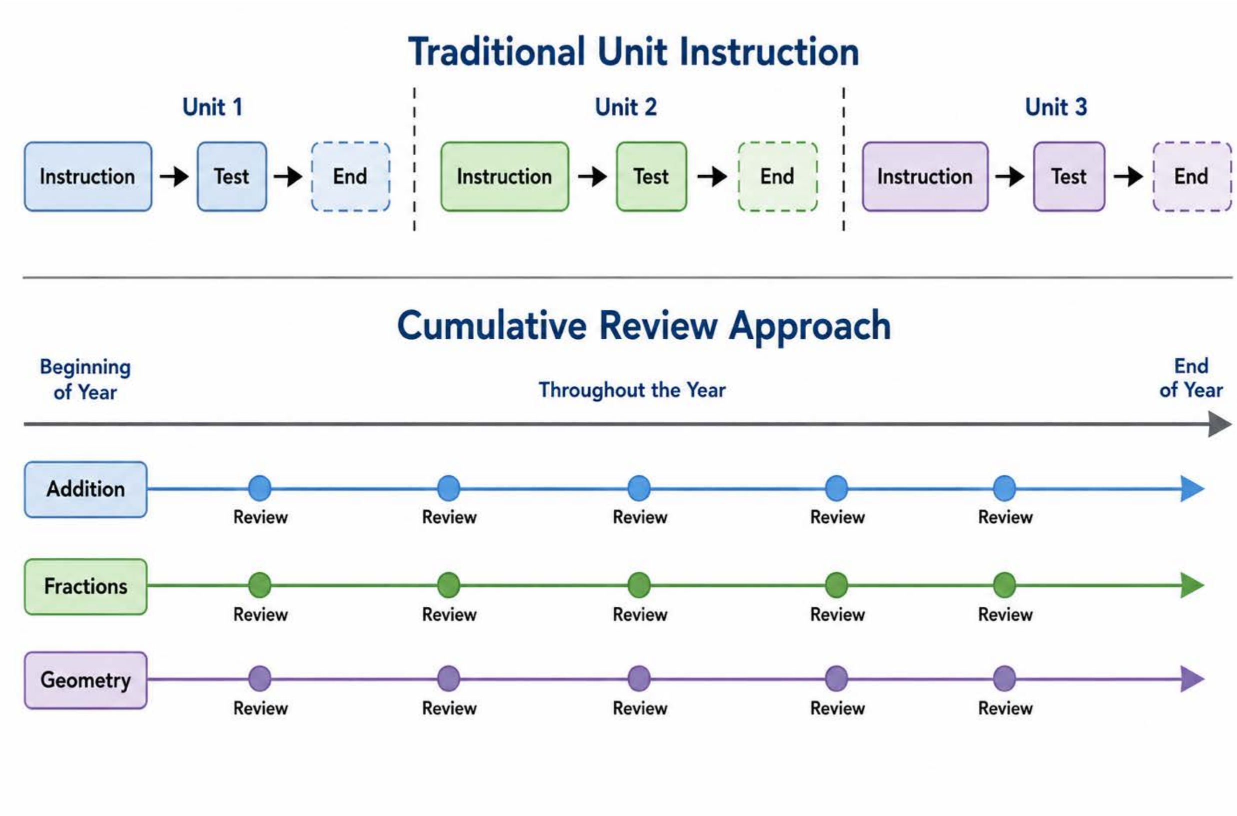


Research Foundation: Cumulative Review

Cumulative review is an instructional practice in which previously learned concepts and skills continue to be revisited throughout the learning process rather than being addressed only during a single instructional unit. By providing ongoing opportunities for students to engage with prior learning, cumulative review helps strengthen retention, reinforce understanding, and reduce the likelihood of skill loss over time. Research suggests that students are more likely to maintain access to important knowledge when previously learned content is systematically reviewed and applied across multiple learning opportunities (Dunlosky et al., 2013).

Traditional instructional approaches often organize learning into discrete units that conclude with an assessment before moving to new content. While this structure may support short-term performance, students frequently experience difficulty retaining information when concepts are no longer revisited. As a result, previously mastered skills may become less accessible over time, creating gaps that interfere with future learning.

Figure 4. *Traditional Unit Instruction Compared with Cumulative Review*



Note. Figure created by Frog Publications, Inc. based on concepts discussed in Ebbinghaus (1885/1913), National Mathematics Advisory Panel (2008), and Dunlosky et al. (2013).

Rosenshine (2012) emphasized the importance of review as a component of effective instruction. His principles of instruction include daily review, weekly review, and periodic opportunities for students to revisit previously learned material. According to Rosenshine (2012), systematic review helps strengthen retention and increases the likelihood that students will successfully connect prior learning with new concepts.

The importance of cumulative review is particularly evident in mathematics education. Mathematical concepts are interconnected, and success with advanced skills often depends upon mastery of foundational knowledge. Students who lose access to previously learned concepts may struggle when those concepts serve as prerequisites for new learning. Ongoing review helps maintain access to essential knowledge and supports the development of mathematical proficiency over time (National Mathematics Advisory Panel, 2008).

The instructional design of *Drops in the Bucket*® mathematics incorporates cumulative review throughout the program. Previously taught concepts continue to appear within daily practice activities, allowing students to revisit and apply mathematical knowledge long after initial instruction has occurred. Rather than limiting review to occasional remediation opportunities, the program embeds cumulative review into regular instructional practice.

This ongoing review process encourages students to maintain access to previously learned concepts while strengthening connections among mathematical ideas. Through repeated opportunities to revisit important skills, students are able to reinforce learning, improve retention, and develop greater confidence in their mathematical abilities.

Research supports the use of cumulative review as a means of promoting durable learning and long-term retention (Dunlosky et al., 2013; Rosenshine, 2012). By incorporating previously learned concepts into ongoing practice opportunities, *Drops in the Bucket*® mathematics aligns with instructional principles associated with improved retention, mathematical fluency, and academic achievement.

Formative Assessment and Progress Monitoring

Formative assessment is an instructional process used to gather information about student learning during instruction in order to inform educational decisions and improve learning outcomes. Unlike summative assessments, which are typically administered after instruction has concluded, formative assessment provides ongoing feedback that allows educators to identify strengths, address learning gaps, and adjust instruction to meet student needs (Black & Wiliam, 1998).

Research has demonstrated that formative assessment can have a significant positive impact on student achievement when assessment information is used to guide instructional decision making. Black and Wiliam (1998) concluded that effective formative assessment practices provide educators with valuable insights into student understanding and support efforts to improve learning outcomes. By monitoring student progress regularly, educators are better positioned to provide timely interventions and targeted instructional support.

In mathematics education, formative assessment plays an important role in identifying misconceptions, monitoring skill development, and determining whether students have mastered essential concepts. Because mathematical learning is cumulative, difficulties with foundational skills may affect a student's ability to succeed with more advanced content. Ongoing assessment allows educators to identify these challenges early and implement appropriate instructional responses before learning gaps become more significant.

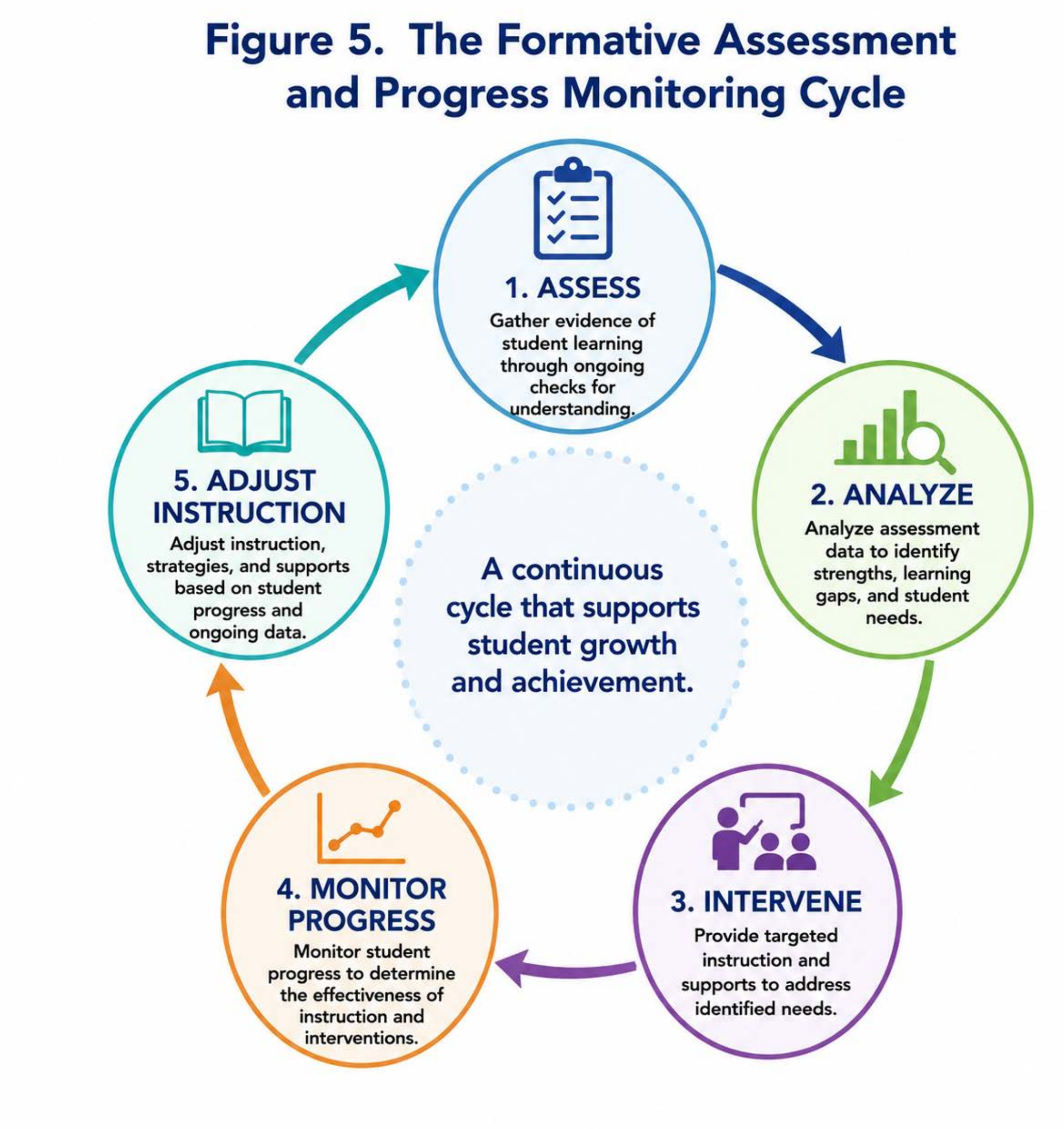
The instructional design of *Drops in the Bucket*® mathematics supports formative assessment through the inclusion of assessment and progress-monitoring resources that provide educators with information about student performance and growth. These tools allow teachers to evaluate mastery of mathematical skills, monitor student progress over time, and identify areas requiring additional instruction or intervention.

Progress-monitoring practices are particularly important within Multi-Tiered Systems of Support (MTSS), where instructional decisions are informed by student performance data. Regular assessment enables educators to determine the effectiveness of interventions, evaluate student growth, and make informed decisions regarding instructional intensity and support. The use of progress-monitoring data can help ensure that students receive appropriate levels of instruction based on demonstrated needs.

By combining ongoing assessment opportunities with systematic review and practice, *Drops in the Bucket*® mathematics provides educators with tools that support data-informed instruction and continuous improvement. These assessment practices align with research-supported approaches that emphasize the use of evidence to guide instructional decisions and improve student outcomes (Black & Wiliam, 1998).

Formative assessment and progress monitoring serve as important components of effective mathematics instruction. Through the collection and analysis of student performance data, educators can better understand student needs, target instructional support, and promote continued mathematical growth and achievement.

Figure 5. *The Formative Assessment and Progress Monitoring Cycle*



Note. Figure created by Frog Publications, Inc. based on concepts discussed in Black and Wiliam (1998) regarding formative assessment and the use of assessment information to guide instructional decision making.

Multi-Tiered Systems of Support (MTSS) and Intervention Framework

Multi-Tiered Systems of Support (MTSS) is a framework designed to provide students with varying levels of instructional support based on demonstrated academic and behavioral needs. Within an MTSS framework, educators use assessment data, progress-monitoring information, and evidence-based instructional practices to ensure that students receive appropriate levels of support and intervention. The goal of MTSS is to improve student outcomes through early identification of learning needs, targeted intervention, and ongoing monitoring of progress (National Mathematics Advisory Panel, 2008).

Effective MTSS implementation relies upon the use of data-informed instructional decision making. Educators regularly collect and analyze student performance information to determine whether students are responding successfully to instruction and intervention efforts. This process allows schools to provide additional support when necessary while ensuring that instructional resources are used efficiently and effectively.

Mathematics instruction is particularly well suited to an MTSS approach because mathematical learning develops progressively over time. Difficulties with foundational concepts can affect future learning and contribute to widening achievement gaps if not addressed promptly. Early identification of learning needs, combined with targeted instructional support, can help students develop the skills necessary for continued success in mathematics (National Mathematics Advisory Panel, 2008).

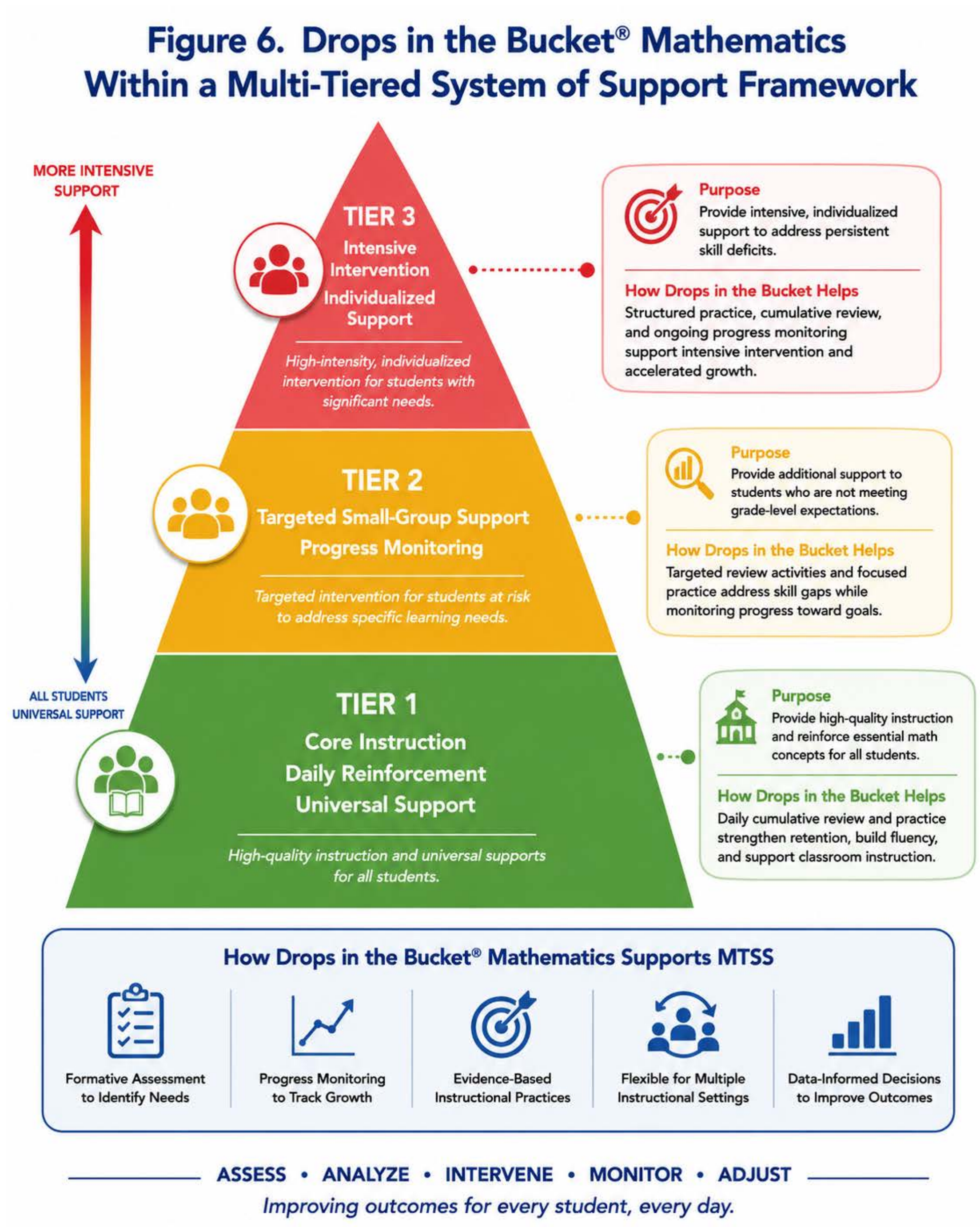
Drops in the Bucket® mathematics supports implementation within a Multi-Tiered System of Support through its flexible instructional design, assessment resources, and progress-monitoring tools. The program may be used to reinforce core classroom instruction, provide targeted intervention, and support intensive remediation efforts across multiple instructional settings.

At the Tier 1 level, *Drops in the Bucket*® mathematics provides systematic review and reinforcement of grade-level mathematical concepts through daily practice opportunities. Consistent exposure to previously taught skills helps support retention, strengthen procedural fluency, and reinforce classroom instruction for all students.

At the Tier 2 level, educators may use assessment and progress-monitoring information to identify students requiring additional support. Targeted review activities and focused practice opportunities can help address specific skill deficits while providing students with additional exposure to important mathematical concepts.

At the Tier 3 level, the program can support intensive intervention efforts by providing structured opportunities for repeated practice, cumulative review, and ongoing progress monitoring. Educators may use student performance data to individualize instruction and monitor growth over time.

Figure 6. *Drops in the Bucket Mathematics Within a Multi-Tiered System of Support Framework*



Note. Figure created by Frog Publications, Inc. to illustrate potential applications of *Drops in the Bucket*® mathematics within a Multi-Tiered System of Support framework.

The integration of assessment, progress monitoring, and systematic review supports data-informed decision making throughout the intervention process. By providing educators with tools to monitor student performance and adjust instruction accordingly, *Drops in the Bucket*® mathematics aligns with instructional practices commonly associated with effective MTSS implementation.

Through flexible implementation options and support for multiple levels of instructional intensity, *Drops in the Bucket*® mathematics helps educators address diverse learning needs while promoting continued mathematical growth and achievement.

Mathematical Thinking and Problem Solving

Mathematical proficiency extends beyond the ability to perform calculations accurately. Effective mathematics instruction also develops students' capacity to reason, solve problems, recognize patterns, make connections among concepts, and apply mathematical knowledge in meaningful situations. Research suggests that students develop deeper mathematical understanding when they engage in learning experiences that require both procedural fluency and conceptual understanding (National Mathematics Advisory Panel, 2008).

The National Mathematics Advisory Panel (2008) identified conceptual understanding, procedural fluency, and problem-solving ability as essential components of mathematical proficiency. Students who develop strong mathematical understanding are better able to explain their thinking, select appropriate solution strategies, and apply previously learned concepts to unfamiliar situations. These abilities support success not only in mathematics but also in future academic and real-world problem-solving contexts.

Mathematical reasoning involves the ability to analyze information, identify relationships, evaluate possible solutions, and justify conclusions. As students encounter increasingly complex mathematical concepts, they must draw upon prior knowledge while determining which strategies and procedures are most appropriate for a given task. Opportunities to engage in reasoning and decision making help students move beyond memorization toward deeper understanding of mathematical ideas.

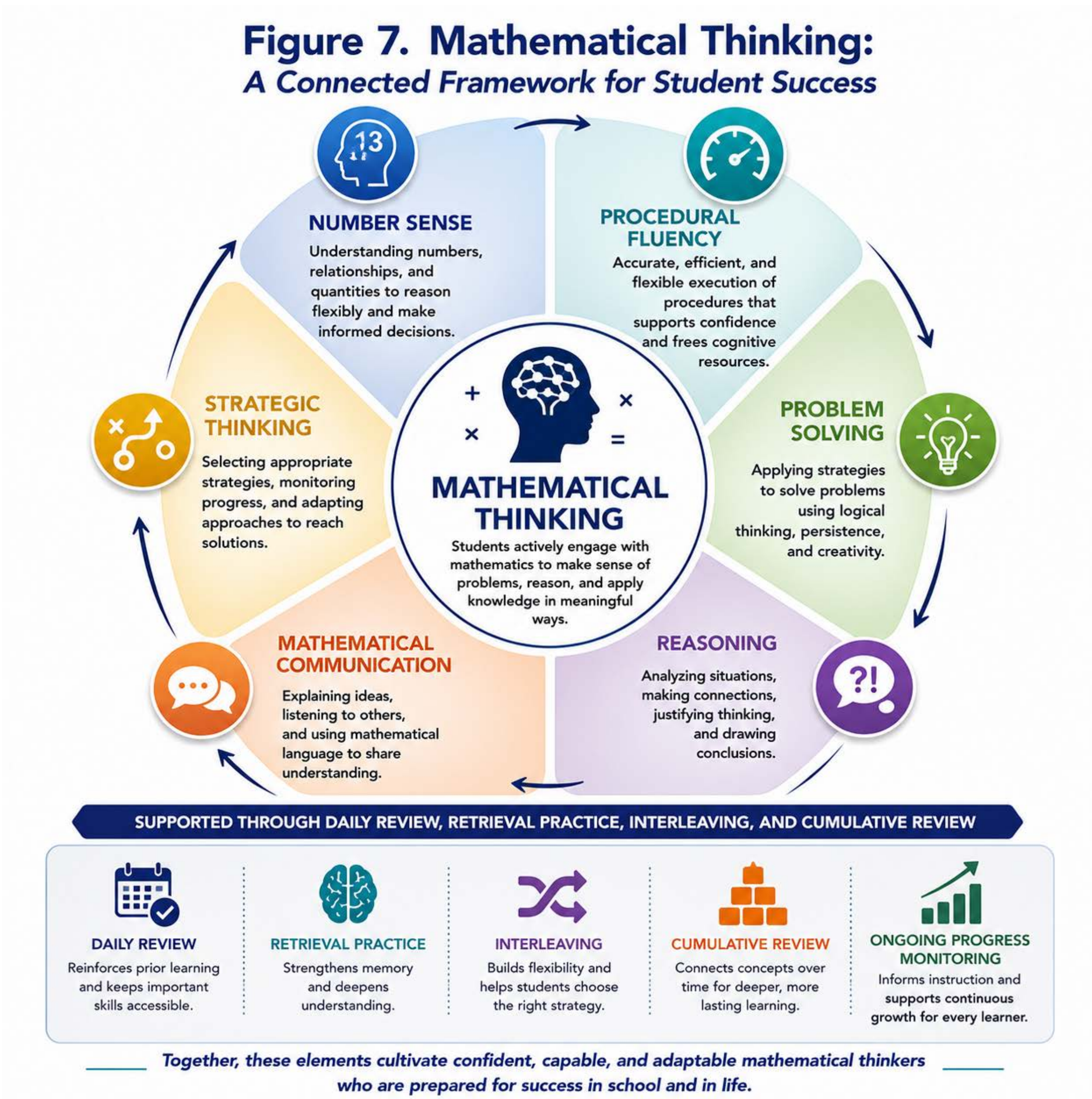
The instructional design of *Drops in the Bucket*® mathematics supports mathematical thinking through the integration of varied problem types, cumulative review, retrieval practice, and interleaved learning opportunities. Because students regularly encounter different mathematical concepts within a single lesson, they must identify relevant information, select appropriate strategies, and apply previously learned knowledge across multiple contexts. These experiences encourage students to think critically about mathematical relationships rather than relying solely on repetitive procedures.

The program's daily review structure also supports the development of mathematical reasoning by requiring students to continually connect current learning with previously learned concepts. As students revisit skills over time, they strengthen their understanding of how mathematical ideas relate to one another and develop greater flexibility in their problem-solving approaches. This process encourages students to recognize patterns, make connections, and apply learning in increasingly sophisticated ways.

Problem solving is further supported through opportunities for students to engage with a variety of mathematical tasks and situations. Rather than focusing exclusively on isolated skill practice, students encounter mathematical experiences that require them to analyze problems, determine appropriate strategies, and evaluate solutions. These opportunities help develop confidence, persistence, and strategic thinking while supporting long-term mathematical growth.

Research indicates that learning experiences requiring active engagement, retrieval of prior knowledge, and application of concepts contribute to stronger retention and improved transfer of learning (Dunlosky et al., 2013). By incorporating these instructional principles into daily practice opportunities, *Drops in the Bucket*® mathematics supports the development of mathematical reasoning, problem solving, and conceptual understanding alongside procedural fluency.

Figure 7. *Mathematical Thinking: A Connected Framework for Student Success*



Note. Figure created by Frog Publications, Inc. to illustrate how daily review, retrieval practice, spaced practice, interleaving, cumulative review, and ongoing progress monitoring support the development of mathematical thinking, problem solving, and reasoning. The framework is informed by concepts discussed in the National Mathematics Advisory Panel (2008) and Dunlosky et al. (2013).

Mathematical thinking develops through repeated opportunities to reason, analyze, connect, and apply knowledge. Through its emphasis on daily review, cumulative learning, varied problem types, and ongoing reinforcement, *Drops in the Bucket*® mathematics provides students with opportunities to strengthen the reasoning and problem-solving skills necessary for long-term mathematical success.

Theory of Action

A theory of action describes the relationship between instructional practices and the outcomes they are intended to produce. It provides a research-informed explanation of how and why a program is expected to improve student learning. The theory of action underlying *Drops in the Bucket*® mathematics is based upon established principles of learning science, mathematics education, and evidence-based instructional practice.

The instructional design of *Drops in the Bucket*® mathematics is grounded in the understanding that mathematical proficiency develops through repeated opportunities to review, retrieve, apply, and connect previously learned knowledge. Research suggests that students are more likely to retain and successfully apply mathematical concepts when learning opportunities incorporate retrieval practice, spaced review, interleaving, cumulative review, and ongoing assessment (Dunlosky et al., 2013; National Mathematics Advisory Panel, 2008).

The program is designed to provide students with consistent, structured opportunities to engage with mathematical concepts over time. Through daily review and reinforcement, students repeatedly access previously learned knowledge, strengthen memory, and maintain important mathematical skills. The inclusion of varied problem types encourages students to select appropriate strategies, make connections among concepts, and apply learning across multiple contexts.

Assessment and progress-monitoring components provide educators with information that can be used to guide instructional decisions and support student growth. When combined with systematic review and intervention opportunities, these practices help ensure that students receive the instructional support necessary to address learning needs and build mathematical proficiency.

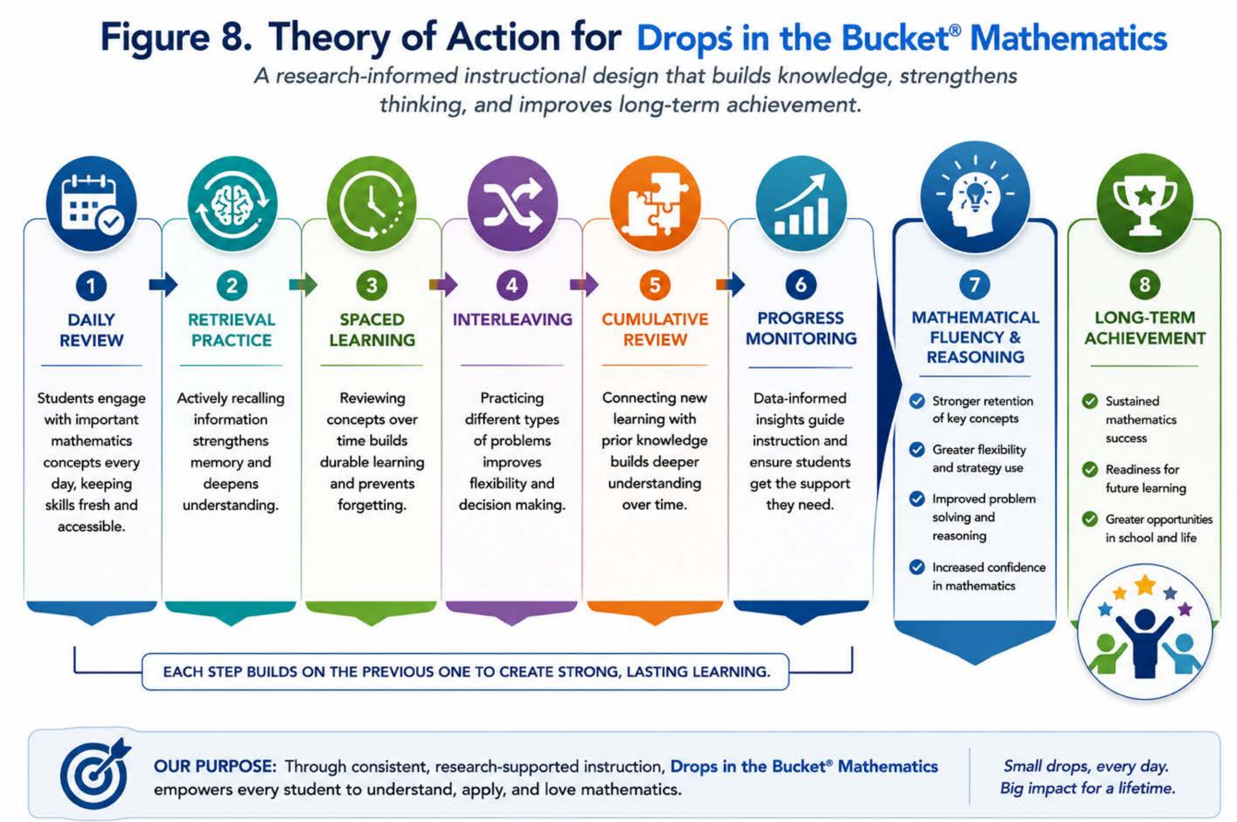
The theory of action for *Drops in the Bucket®* mathematics may be summarized as follows:

If students engage in consistent daily mathematics review that incorporates retrieval practice, spaced learning opportunities, interleaving, cumulative review, and ongoing progress monitoring, then they will strengthen retention of previously learned concepts, develop greater mathematical fluency and reasoning, maintain access to foundational skills, and improve their ability to apply mathematical knowledge in new situations.

As students experience these research-supported learning opportunities over time, they are expected to demonstrate stronger mathematical understanding, increased confidence, improved problem-solving abilities, and greater readiness for future mathematical learning.

Through the integration of evidence-based instructional practices and ongoing opportunities for reinforcement, *Drops in the Bucket®* mathematics is designed to support long-term mathematical growth and achievement for a diverse range of learners and instructional settings.

Figure 8. *Theory of Action for Drops in the Bucket Mathematics*



Note. Figure created by Frog Publications, Inc. to illustrate the theory of action underlying *Drops in the Bucket®* mathematics. The model demonstrates how daily review, retrieval practice, spaced learning, interleaving, cumulative review, progress monitoring, and mathematical reasoning work together to support long-term mathematical achievement. The framework is informed by concepts discussed in Ebbinghaus (1885/1913), Cepeda et al. (2006), Roediger and Karpicke (2006), Dunlosky et al. (2013), the National Mathematics Advisory Panel (2008), and Black and Wiliam (1998).

ESSA Alignment and Evidence-Based Practices

The Every Student Succeeds Act (ESSA) emphasizes the use of evidence-based instructional practices and interventions to improve student achievement. ESSA defines evidence-based activities, strategies, and interventions as those that demonstrate a statistically significant effect on improving student outcomes or are supported by a strong research-based rationale. Educational programs that incorporate research-supported instructional principles may assist schools in meeting ESSA expectations for evidence-informed decision making (Every Student Succeeds Act, 2015).

The instructional design of *Drops in the Bucket®* mathematics reflects multiple research-supported practices identified in educational and cognitive science literature. The program incorporates retrieval practice, spaced learning opportunities, interleaving, cumulative review, formative assessment, and progress monitoring. These instructional practices have been associated with improved retention, stronger transfer of learning, enhanced mathematical reasoning, and increased academic achievement (Dunlosky et al., 2013; National Mathematics Advisory Panel, 2008).

ESSA recognizes the importance of using instructional approaches grounded in research and supported by evidence. The research foundations underlying *Drops in the Bucket®* mathematics are consistent with evidence-based principles that have been widely discussed within mathematics education, cognitive psychology, and instructional design research. These principles support effective teaching and learning by promoting retention, understanding, and application of mathematical knowledge.

The program's flexible implementation model allows educators to incorporate evidence-based instructional practices within a variety of educational settings. *Drops in the Bucket*® mathematics may be used to reinforce Tier 1 instruction, support targeted interventions, provide additional learning opportunities for struggling students, and supplement enrichment activities for students requiring additional challenges.

Assessment and progress-monitoring components further support evidence-informed instructional decision making. By providing educators with tools to monitor student growth and identify learning needs, the program helps support data-driven instructional planning and intervention efforts consistent with MTSS frameworks and school improvement initiatives.

The research-supported principles incorporated throughout *Drops in the Bucket*® mathematics provide a strong rationale for its use as part of a comprehensive mathematics instructional framework. Through the integration of evidence-based practices and ongoing opportunities for reinforcement and review, the program supports instructional goals associated with long-term mathematical achievement and student success.

While individual implementation outcomes may vary across settings, the instructional design of *Drops in the Bucket*® mathematics reflects a research-based rationale supported by established learning science principles and mathematics education research. These characteristics align with ESSA's emphasis on evidence-informed educational practices designed to improve student learning outcomes.

Figure 9. *Alignment of Drops in the Bucket Mathematics*

Note. Figure created by Frog Publications, Inc. to illustrate the alignment of *Drops in the Bucket®* mathematics with research-supported instructional practices commonly associated with evidence-informed educational programs.

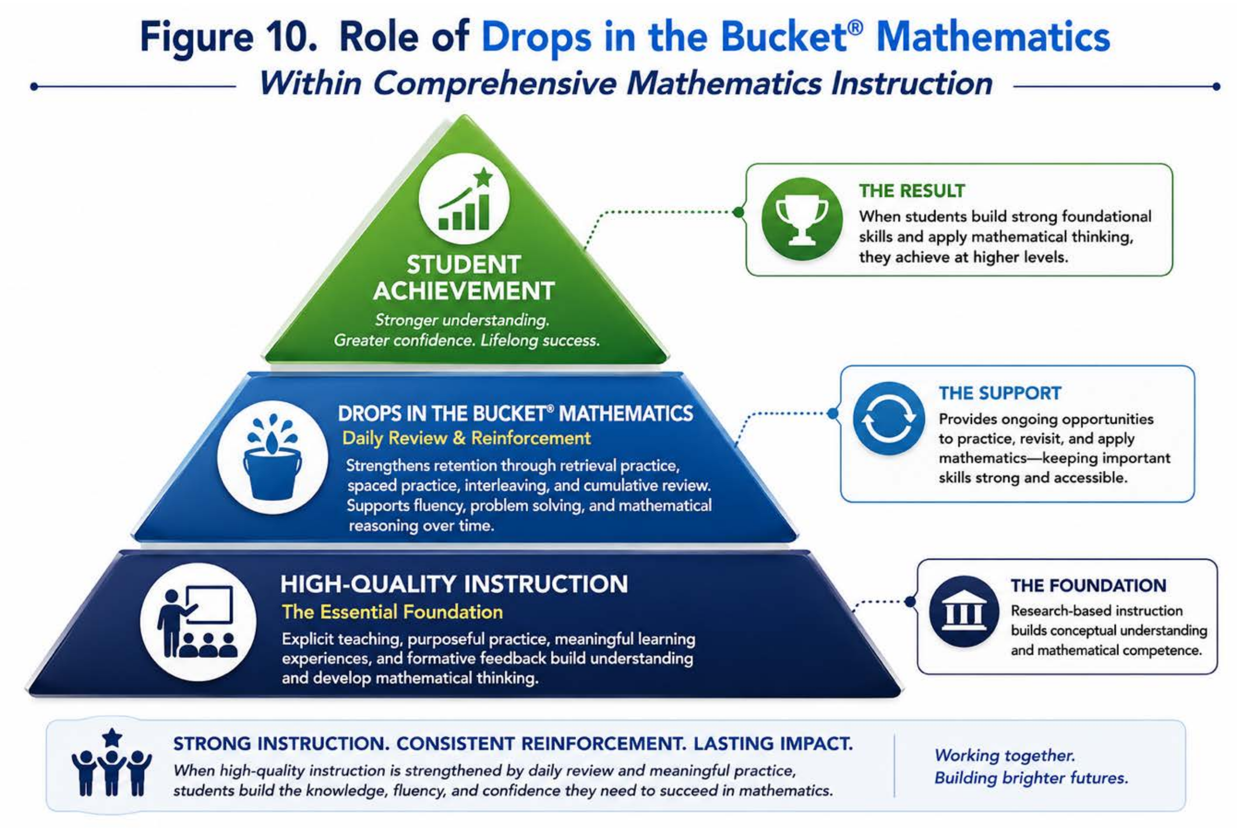
Appropriate Use and Implementation Considerations

Effective mathematics instruction involves the integration of high-quality teaching, meaningful learning experiences, ongoing assessment, and opportunities for practice and reinforcement. Research suggests that students benefit from instructional environments that combine explicit instruction, guided practice, independent application, feedback, and systematic review of previously learned concepts (National Mathematics Advisory Panel, 2008).

Drops in the Bucket® mathematics is designed to support and reinforce mathematics instruction through daily review, retrieval practice, cumulative review, and ongoing opportunities for skill application. The program is intended to complement classroom instruction by helping students maintain access to previously learned knowledge and strengthen long-term retention of important mathematical concepts.

The program is not intended to replace comprehensive mathematics curricula, teacher-led instruction, or standards-based mathematics lessons. Rather, the program is designed to function as a reinforcement and review system that can be integrated into a variety of instructional settings and program models.

Figure 10. *Role of Drops in the Bucket Mathematics*



Note. Figure created by Frog Publications, Inc. to illustrate how *Drops in the Bucket®* mathematics complements high-quality mathematics instruction through the use of daily review, retrieval practice, spaced learning, interleaving, cumulative review, and progress monitoring. The model reflects research-supported instructional principles discussed throughout this document and is intended to demonstrate the program's role in supporting long-term mathematical growth and achievement.

Because schools differ in instructional priorities, schedules, student populations, and intervention frameworks, implementation may vary across classrooms and educational settings.

Educators are encouraged to use professional judgment when determining how *Drops in the Bucket*® mathematics can best support local instructional goals and student learning needs.

The flexible design of the program allows it to be incorporated into core instruction, intervention settings, enrichment opportunities, tutoring programs, after-school learning experiences, and summer learning initiatives. Assessment and progress-monitoring tools may be used to support instructional planning and data-informed decision making when appropriate.

The effectiveness of any instructional resource depends upon thoughtful implementation, alignment with student needs, and the quality of instruction provided. When used as part of a comprehensive instructional approach, *Drops in the Bucket*® mathematics provides educators with a research-informed tool for reinforcing learning, strengthening retention, and supporting long-term mathematical growth.

Conclusion

Mathematics achievement depends upon students' ability to retain, apply, and build upon previously learned knowledge over time. Research in cognitive science, mathematics education, and instructional design has consistently demonstrated that durable learning develops through repeated opportunities to retrieve, review, connect, and apply important concepts. Effective mathematics instruction supports these processes by providing students with meaningful opportunities to strengthen understanding, maintain access to foundational skills, and engage in ongoing mathematical thinking.

The instructional design of *Drops in the Bucket*® mathematics reflects multiple research-supported principles associated with effective learning and long-term retention. Through the integration of retrieval practice, spaced learning, interleaving, cumulative review, formative assessment, progress monitoring, and support for mathematical reasoning, the program provides students with structured opportunities to revisit and reinforce important mathematical concepts over time.

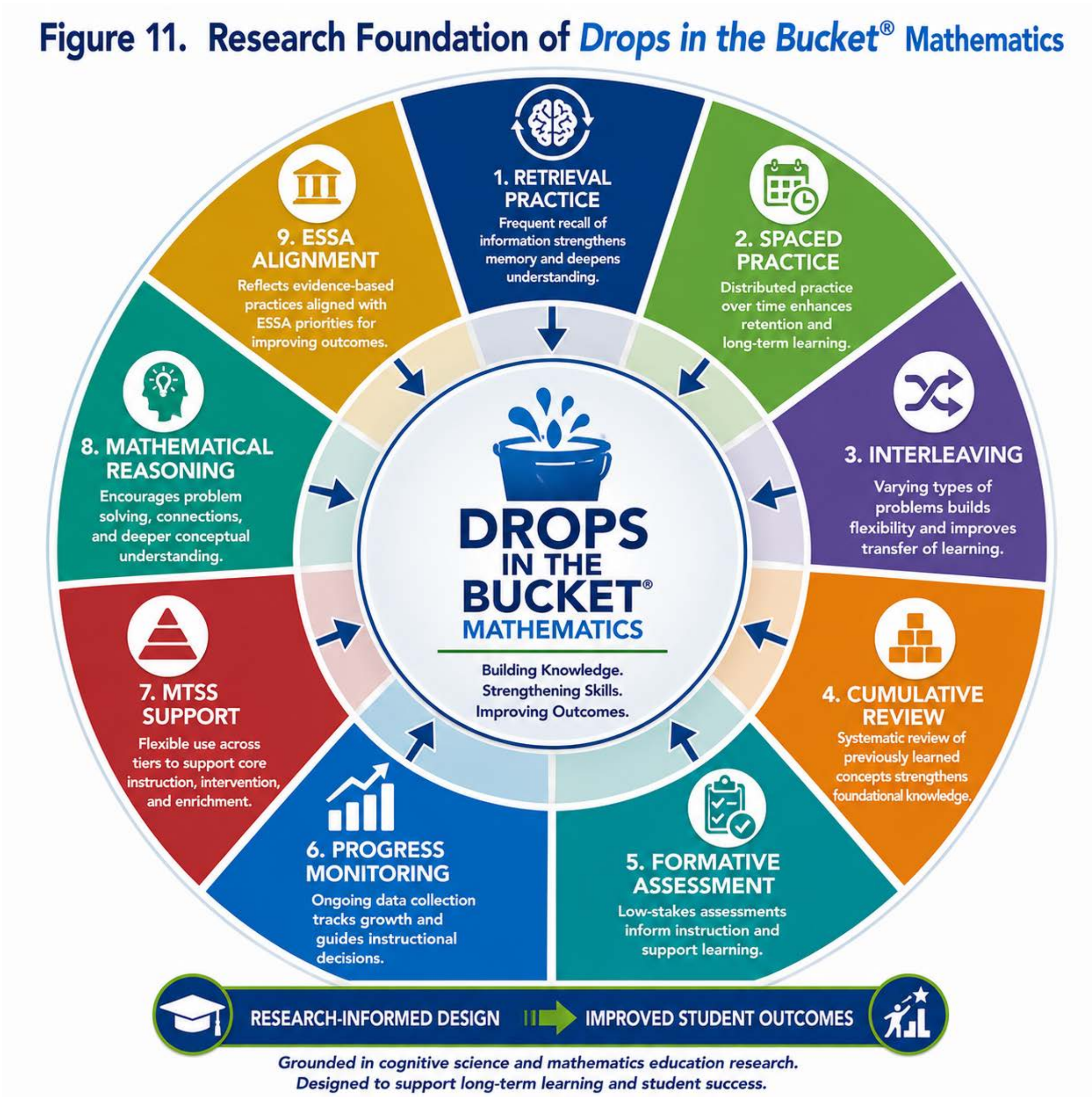
The program's emphasis on daily review recognizes that mathematical learning is cumulative and that previously learned skills must remain accessible if students are to successfully engage with increasingly complex concepts. By incorporating varied problem types, ongoing reinforcement, and opportunities for application, *Drops in the Bucket*® mathematics encourages students to strengthen procedural fluency, develop conceptual understanding, and improve problem-solving abilities.

The research reviewed throughout this document provides a strong rationale for instructional approaches that emphasize systematic review, active retrieval, distributed learning opportunities, and data-informed decision making. These principles align with evidence-based practices commonly discussed within mathematics education, Multi-Tiered Systems of Support (MTSS), and the Every Student Succeeds Act (ESSA) framework.

Drops in the Bucket® mathematics is designed to complement high-quality instruction by providing educators with a flexible, research-informed tool for reinforcing learning and supporting student growth. The program may be implemented across a variety of instructional settings, including core instruction, intervention, enrichment, tutoring, after-school programs, and summer learning opportunities.

Ultimately, the goal of *Drops in the Bucket*® mathematics is to help students build and maintain the mathematical knowledge, skills, confidence, and reasoning abilities necessary for continued academic success. Through consistent reinforcement of important concepts and alignment with research-supported instructional principles, the program seeks to support long-term mathematical achievement for all learners.

Figure 11. *Research Foundation of Drops in the Bucket Mathematics*



Note. Figure created by Frog Publications, Inc. to summarize the research foundation underlying *Drops in the Bucket*® mathematics. The model illustrates the integration of retrieval practice, spaced practice, interleaving, cumulative review, formative assessment, progress monitoring, MTSS support, mathematical reasoning, and evidence-based instructional principles that collectively support long-term mathematical achievement and student success.

About Frog Publications

Founded in 1975, Frog Publications is an educational publisher dedicated to developing instructional materials that support skill development, retention, and academic achievement. For more than 51 years, the company has provided educators with practical resources designed to reinforce learning through meaningful practice and ongoing review.

Its materials are used in classrooms, intervention programs, tutoring settings, after-school initiatives, summer learning opportunities, and home learning environments across the United States. Guided by research-informed instructional principles and a commitment to educational excellence, Frog Publications continues to develop resources that help students build confidence, strengthen foundational skills, and achieve long-term academic success.

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